Indian Statistical Institute, Bangalore Centre
B.Math. (I Year): 2008-2009
Semester II: Backpaper Examination
Probability Theory - II

Time : 3 hrs. Maximum marks : 100

1. [12+6+6 marks] Let

$$f(x,y) = C\exp\{-\frac{1}{2}(x^2 - xy + 4y^2)\}, \quad (x,y) \in \mathbb{R}^2.$$

(i) Find C so that f is a bivariate probability density function.
(ii) Find the marginal density functions.
(iii) Find the conditional density functions.

2. [10 marks] Let $X_1, X_2, \ldots, X_n$ be random variables having finite second moments. Let $\Sigma = ((\sigma_{ij})), 1 \leq i,j \leq n$ be the real $(n \times n)$ matrix given by $\sigma_{ij} = \mathrm{Cov}(X_i, X_j)$. Show that Σ is a symmetric nonnegative definite matrix.

3. [13+3 marks] Let $X_1, X_2, \ldots, X_n$ be independent $N(0,1)$ random variables. Let $Y_k = X_1 + X_2 + \cdots + X_k$, $k = 1, 2, \ldots, n$.
(i) Find the distribution of $(Y_1, Y_2, \ldots, Y_n)$.
(ii) Find $\mathrm{Cov}(Y_i, Y_j)$.

4. [15 marks] Let X have $N(\mu, \sigma^2)$ distribution. Find $E(\cos(tX))$, $t \in \mathbb{R}$.

5. [20 marks] 200 numbers are rounded off to the nearest integer and then added. Assume that the round off errors are independent and uniformly distributed over $(-\frac{1}{2}, \frac{1}{2})$. Find the probability that the computed sum will differ from the sum of the original 200 numbers by more than 5.

6. [15 marks] Using characteristic functions show that the sum of independent Poisson random variables is again Poisson.